

# $\mathcal{N} = 2$ SCFTs: An M5-brane perspective

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## Abstract

Inspired by the recently discovered holographic duality between  $\mathcal{N} = 2$  SCFTs and half-BPS M-theory backgrounds, we study probe M5-branes. Though our main focus is supersymmetric M5-branes whose worldvolume has an  $AdS_n$  factor, we also consider some other configurations. Of special mention is the identification of  $AdS_5$  and  $AdS_3$  probes preserving supersymmetry, with only the latter supporting a self-dual field strength.

# 1 Introduction

Four-dimensional  $\mathcal{N} = 2$  supersymmetric theories are truly remarkable. Compared to  $\mathcal{N} = 4$  supersymmetric theories, which are finite, they are much richer in physics, but yet still solvable. Especially, they provide a window to study the nonperturbative aspects of quantum field theories and have been widely studied for more than fifteen years since Seiberg and Witten's monumental works [1, 2]. However, the surprises they present to us have not come to the end. Recent developments on four-dimensional  $\mathcal{N} = 2$  superconformal theories starting from [3] have drawn lots of attention. This class of generalized quiver theories could be constructed geometrically by wrapping M5 branes on Riemann surfaces with genus and punctures. The electric-magnetic duality and the Argyres-Seiberg duality [4] have since been generalized to these theories. It turns out that the gauge couplings of the theory are encoded in the complex structure moduli of the Riemann surface, including the position of the punctures. More interestingly, it was conjectured in [5] that the Nekrasov partition function of these theories with  $SU(2)$  gauge groups could be related to the conformal blocks and correlation functions of the Liouville theory. Non-local operators in these four-dimensional theories have been studied in [6, 7, 8, 9, 10] (See also [11]).

The holographic dual of these theories in the large  $N$  limit was neatly studied in [12]. The theory on the gravity side of this AdS/CFT correspondence is M-theory on backgrounds which are the products of  $AdS_5$  spacetime and six-dimensional internal manifolds with  $SU(2) \times U(1)$  isometry. These gravity backgrounds belong to the general geometries found in [13]. Of particular interest among these backgrounds is the so-called Maldacena-Núñez(MN) geometry, which was first discovered in [14] by considering the IR limit of M5-branes wrapped on a Riemann surface. In this case, the six-dimensional internal manifold is simply an  $S^4$  fibered over the Riemann surface. On the other side, the field theory corresponding to the MN geometry comes from M5-branes wrapping the same Riemann surface. It is remarkable, that in this case, there are no punctures on the Riemann surface and that the building block of the quiver gauge theory is a strongly coupled superconformal theory  $T_N$  with three  $SU(N)$  global symmetries, yet without a coupling constant. One motivation of this work is to understand this intriguing  $T_N$  theory from its gravity dual.

Probe branes play an important role in the gravity side of holographic correspondences. These branes are intrinsically stringy but accessible. Among other things, they could be dual to local operators [15, 16, 17], loop operators [18, 19, 20], surface operators [21, 22], or domain walls [23] in the field theory side of various AdS/CFT correspondences. Adding suitable branes can also add flavor to the field theory [24].

In this paper, we plan to start the search for interesting probe M5-branes in the LLM geometries studied in [12]. We mainly focus on the simplest MN background which we

have rederived in the appendix to include the fluxes. The form of the solution is

$$\begin{aligned}
ds_{11}^2 &= \tilde{\kappa}^{2/3} \left( \frac{1}{2} W^{1/3} ds_{AdS_5}^2 + \frac{W^{-2/3}}{4} \left[ W \frac{(dx^2 + dy^2)}{y^2} \right. \right. \\
&\quad \left. \left. + W d\theta^2 + \cos^2 \theta (d\phi_1^2 + \sin^2 \phi_1 d\phi_2^2) + 2 \sin^2 \theta \left( d\chi + \frac{dx}{y} \right)^2 \right] \right), \\
H_4 &= \tilde{\kappa} \left( -\frac{1}{4W^2} [3 + \cos^2 \theta] \sin \theta \cos^2 \theta d\theta \left( d\chi + \frac{dx}{y} \right) + \frac{1}{4} \frac{\cos^3 \theta}{W} \frac{dx dy}{y^2} \right) d^2 \Omega,
\end{aligned} \tag{1}$$

where  $\phi_1, \phi_2$  parameterize a two-sphere,  $x, y$  denote a hyperbolic Riemann surface  $\Sigma_2$  of constant negative curvature, and  $W$  is

$$W = 1 + \cos^2 \theta. \tag{2}$$

The constant  $\tilde{\kappa}$  denotes an additional scale factor that will be accounted for by ensuring that the flux is correctly quantised.<sup>1</sup>

This geometry may also be obtained from the most general solutions of eleven dimensional supergravity preserving  $\mathcal{N} = 2$  superconformal symmetry [13] as described in [12]. Although we focus on the above background, we believe our results can be generalized to more general LLM geometries. In the literature, some BPS probe branes have been studied in [12], and half-BPS M2-brane dual to loop operator have appeared in [6].

Starting with Killing spinors of MN geometry and kappa symmetry for M5, one can search for BPS M5-branes. As a first step, one needs to determine the Killing spinors preserved by the MN solution. Luckily, this has already been done for the most general solution [13] and the Killing spinors corresponding to the analytically continued solution corresponding to MN have appeared in [6]. The latter appears without derivation, so in the appendix we validate their claim by following a similar decomposition to that appearing in [13] (see also [25])<sup>2</sup>. The result of that exercise is that the eleven-dimensional Killing spinors of the MN solution can be expressed in terms of  $AdS_5$  ( $\psi$ ) and  $S^2$  ( $\chi_+$ ) Killing spinors as

$$\begin{aligned}
\epsilon &= e^{\lambda/2} \psi \otimes (1 + i\sigma_3 \otimes \gamma_{(4)}) \chi_+ \otimes e^{-\frac{i}{2}\phi_0 \gamma_{10}} e^{i\chi/2} \epsilon_0, \\
\epsilon^c &= e^{\lambda/2} \psi^c \otimes (1 - i\sigma_3 \otimes \gamma_{(4)}) \chi_+ \otimes e^{-\frac{i}{2}\phi_0 \gamma_{10}} e^{-i\chi/2} \gamma_7 \epsilon_0,
\end{aligned} \tag{3}$$

where the superscript  $c$  denotes the conjugate and

$$\sin \phi_0 = \frac{\sqrt{2} \cos \theta}{\sqrt{W}}, \quad \cos \phi_0 = -\frac{\sin \theta}{\sqrt{W}}, \quad e^{2\lambda} = \frac{\tilde{\kappa}^{2/3} W^{1/3}}{8}. \tag{4}$$

<sup>1</sup>It is related to the  $\kappa$  in [12] by  $\tilde{\kappa} = 2^4 N \kappa$ .

<sup>2</sup>We also cure some typos in [6].

The constant spinor  $\epsilon_0$  satisfies the following projection conditions

$$\gamma_9 \epsilon_0 = \epsilon_0, \quad i\gamma_{78} \epsilon_0 = \epsilon_0. \quad (5)$$

With possible dual non-local objects in field theory in our mind, we pay principal attention to M5-branes whose worldvolumes have  $AdS_m$  ( $2 \leq m \leq 5$ ) factors. The brane with worldvolume  $AdS_5 \times S^1$  has been studied previously in [12]. However, we find that although turning on self-dual three-form field strength on the worldvolume in a suitable way does not break supersymmetry, the equations of motion of M5-branes will not be satisfied. This comes as some surprise as the Kaluza-Klein reduction of the two-form potential on the  $S^1(\chi)$  gives rise to a  $U(1)$  gauge field in  $AdS_5$  corresponding to a global symmetry rotating the phase of a dual bifundamental field [12].

Moreover, we find BPS M5-branes with an  $AdS_3$  factor. This brane should be dual to some two-dimensional object in the field theory side. However it is not dual to the supersymmetric surface operator studied in [7, 9], since this brane wraps the Riemann surface in the six-dimensional internal space, instead of intersecting with this Riemann surface at a point. In this case, we find that we can turn on a suitable self-dual three-form field strength on the worldvolume such that the BPS condition and the equations of motion are both satisfied. We also find BPS M5-branes not embedded along the  $AdS_5$  radial direction that satisfy the equations of motion hinting that there should be non-BPS  $AdS$  branes there also. In addition, inspired by some probe M5-branes in  $AdS_7 \times S^4$  [26, 27], we turn to searching for M5-branes in MN background with more complicated worldvolumes. As a result, we find M5-branes with an  $AdS_3 \times S^1$  and  $AdS_2 \times S^2$  factors which are completely embedded in the  $AdS_5$  part of the background. We explicitly illustrate that generically these branes are non-supersymmetric.

In the next section, we move to review the M5-brane equations of motion and BPS condition. With these tools at hand, we study various probe M5-branes in section 3. In section 4, we examine more exotic embeddings in  $AdS_5$  before concluding. Some technical details are located in the appendices.

## 2 M5-brane review

In this section, we review the M5-brane covariant equations of motions in curved spacetime and discuss the condition for the M5-brane probe to preserve supersymmetry. For earlier work on various aspects of the M5-brane, see [29, 30, 31, 32, 33, 34] (For a review of M-theory branes, see [35]). This section echoes the brief review of the M5-brane presented in [27] and we refer the reader there for a further account of the M5-brane action.

Focusing solely on the bosonic components, we simply have two equations of motion: a scalar and a tensor equation. The scalar equation takes the form

$$G^{mn}\nabla_m\mathcal{E}_n^{\underline{c}} = \frac{Q}{\sqrt{-g}}\epsilon^{\mu_1\cdots\mu_6}\left(\frac{1}{6!}H_7^{\underline{a}}{}_{\mu_1\cdots\mu_6} + \frac{1}{(3!)^2}H_4^{\underline{a}}{}_{\mu_1\mu_2\mu_3}H_{\mu_4\mu_5\mu_6}^{\underline{a}}\right)P_{\underline{a}}^{\underline{c}} \quad (6)$$

and the tensor equation is of the form

$$G^{mn}\nabla_m H_{npq} = Q^{-1}(4Y - 2(mY + Ym) + mYm)_{pq}. \quad (7)$$

Here our notation is as follows: indices from the beginning(middle) of the alphabet refer to frame(coordinate) indices, and the underlined indices refer to target space ones. More details of our conventions may be found in the appendices.

Appearing in the equations of motion, we have the following quantities which are defined in terms of the self-dual 3-form field strength  $h$  on the M5-brane worldvolume

$$k_m^{\ n} = h_{mpq}h^{npq}, \quad (8)$$

$$Q = 1 - \frac{2}{3}\text{Tr}k^2, \quad (9)$$

$$m_p^{\ q} = \delta_p^{\ q} - 2k_p^{\ q}, \quad (10)$$

$$H_{mnp} = 4Q^{-1}(1 + 2k)_m^{\ q}h_{qnp} \quad (11)$$

Note that  $h_{mnp}$  is self-dual with respect to worldvolume metric but not  $H_{mnp}$ . The induced metric is simply

$$g_{mn} = \mathcal{E}_m^{\underline{a}}\mathcal{E}_n^{\underline{b}}\eta_{\underline{ab}} \quad (12)$$

where

$$\mathcal{E}_m^{\underline{a}} = \partial_m z^{\underline{m}} E_{\underline{m}}^{\underline{a}}. \quad (13)$$

Here  $z^{\underline{m}}$  is a target spacetime coordinate, which becomes a function of worldvolume coordinate  $\xi$  through the embedding, and  $E_{\underline{m}}^{\underline{a}}$  is the component of target space vielbein. From the induced metric, we can define another tensor

$$G^{mn} = (1 + \frac{2}{3}k^2)g^{mn} - 4k^{mn}. \quad (14)$$

We also have

$$P_{\underline{a}}^{\underline{c}} = \delta_{\underline{a}}^{\underline{c}} - \mathcal{E}_{\underline{a}}^m \mathcal{E}_m^{\underline{c}}. \quad (15)$$

Note that in the scalar equation of motion, the covariant derivative  $\nabla_m \mathcal{E}_n^{\underline{c}}$  involves not only the Levi-Civita connection of the M5-brane worldvolume but also the spin connection of the target spacetime geometry. More precisely, one has

$$\nabla_m \mathcal{E}_n^{\underline{c}} = \partial_m \mathcal{E}_n^{\underline{c}} - \Gamma_{mn}^p \mathcal{E}_p^{\underline{c}} + \mathcal{E}_m^{\underline{a}} \mathcal{E}_n^{\underline{b}} \omega_{\underline{ab}}^{\underline{c}} \quad (16)$$

where  $\Gamma_{mn}^p$  is the Christoffel symbol with respect to the induced worldvolume metric and  $\omega_{\underline{ab}}^{\underline{c}}$  is the spin connection of the background spacetime pulled back to the worldvolume.

Moreover, there is a background 4-form field strength  $H_4{}_{\underline{a}_1\cdots\underline{a}_4}$  and its Hodge dual 7-form  $H_7{}_{\underline{a}_1\cdots\underline{a}_7}$ :

$$\begin{aligned} H_4 &= dC_3 \\ H_7 &= dC_6 + \frac{1}{2}C_3 \wedge H_4 \end{aligned} \quad (17)$$

The frame indices on  $H_4$  and  $H_7$  in the above equations (6) and (7) have been converted to worldvolume indices with factors of  $\mathcal{E}_m^{\underline{c}}$ . From the background fluxes, we can define

$$Y_{mn} = [4 \star \underline{H} - 2(m \star \underline{H} + \star \underline{H}m) + m \star \underline{H}m]_{mn}, \quad (18)$$

where

$$\star \underline{H}^{mn} = \frac{1}{4! \sqrt{-g}} \epsilon^{mnpqrs} \underline{H}_{pqrs} \quad (19)$$

The field  $H_{mnp}$  is defined by

$$H = dA_2 - \underline{C}_3, \quad (20)$$

where  $A_2$  is a 2-form gauge potential and  $\underline{C}_3$  is the pull-back of the bulk gauge potential. From its definition,  $H$  satisfies the Bianchi identity

$$dH = -\underline{H}_4 \quad (21)$$

where  $\underline{H}_4$  is the pull-back of the target space 4-form flux.

In general, the supersymmetric embeddings of a probe brane in a background may be determined from the kappa-symmetry condition

$$\Gamma_\kappa \epsilon = \pm \epsilon. \quad (22)$$

Here,  $\Gamma_\kappa$  denotes the gamma matrix associated to the probe,  $\epsilon$  denotes the Killing spinor of the background and the sign accounts for the choice between brane and anti-brane probes. The amount of unbroken supersymmetry may be determined by keeping track of the additional projection conditions that arise from the above equation.

Specializing to the MN background with M5-brane probes, the kappa symmetry matrix  $\Gamma_{M5}$  may be written following [28]

$$\Gamma_{M5} = \frac{1}{6! \sqrt{-g}} \epsilon^{j_1 \cdots j_6} [\Gamma_{<j_1 \cdots j_6>} + 40 \Gamma_{<j_1 j_2 j_3>} h_{j_4 j_5 j_6}]. \quad (23)$$

Here  $g$  is the determinant of the induced worldvolume metric component,  $h_{j_4 j_5 j_6}$  is the self-dual 3-form on the M5-brane and  $\Gamma_{<j_1 \cdots j_n>}$  is defined as

$$\Gamma_{<j_1 \cdots j_n>} = \mathcal{E}_{j_1}^{\underline{a}_1} \cdots \mathcal{E}_{j_n}^{\underline{a}_n} \Gamma_{\underline{a}_1 \cdots \underline{a}_n}, \quad (24)$$

where  $\Gamma_{\underline{a_1 \dots a_n}}$  is the product of the Gamma matrices in orthonormal frame.

We pause here to make a brief comment. Denoting the worldvolume of the M5 by  $\xi^a, a = 0, \dots, 5$ , in the case of a simple probe configuration, we may rewrite the above projector (22) as

$$[\alpha \Gamma_{012345} + \beta (\Gamma_{012} - \Gamma_{345})] \epsilon = \pm \epsilon, \quad (25)$$

where  $\alpha, \beta$  denote arbitrary factors. Demanding it to be a projector, it is essential the left hand side squares to unity. In that event,  $\beta$  drops out completely and  $\alpha^2 = 1$ , meaning that  $\alpha = \pm 1$ . The implication of this observation, at least for the simple probes considered in this paper, is that if the M5-probe is not supersymmetric, then supersymmetry cannot be restored by introducing  $h$ . So the task in the rest of the paper is pretty straightforward: identify supersymmetric probes and then turn on  $h$  to see if it preserves supersymmetry. At each stage, it is also imperative to ensure that the equations of motion are satisfied.

### 3 Supersymmetric probes

In this section, we focus on the kappa-symmetry condition (22) and isolate probes that will preserve some supersymmetry. We descend in dimension of the part in  $AdS_5$  from  $d=5$  to  $d=2$  and in each case, we enumerate the possibilities. Throughout we differentiate between probes that are  $AdS$  i.e. those incorporating the radial direction  $r$  of  $AdS_5$  and those located at a fixed  $r$ . We begin by examining the  $AdS$  probes.

#### 3.1 Supersymmetric $AdS$ probes

In this subsection, we descend from  $AdS_5$  to  $AdS_2$  and identify the supersymmetric probes (if any), before examining the additional constraints coming from the equations of motion. As a warm-up, we begin with the  $AdS_5$  M5-brane probe which received some attention in [12].

##### $AdS_5$ probes

In general, one can consider studying the probe brane with worldvolume  $AdS_5 \times \mathcal{C}$  in the MN background, where  $\mathcal{C}$  denotes a curve in the six-dimensional space transverse to  $AdS_5$ . We consider the  $\mathcal{C}$  to be parameterised by  $\sigma$  i.e.  $z^m(\sigma)$ . Using  $AdS_5$  Poincaré coordinates, a natural choice for the M5 embedding is

$$\xi_0 = x_0, \quad \xi_i = x^i, \quad \xi_4 = r, \quad \xi_5 = \sigma, \quad (26)$$

where  $i = 1, 2, 3$ .

Adopting the gauge choice  $\sigma = \chi$ , while permitting embeddings of the form  $x \equiv x(\chi), y \equiv y(\chi)$ , the kappa symmetry matrix  $\Gamma_{M5}$  simplifies to

$$\Gamma_{M5} = \frac{1}{\sqrt{g_{\chi\chi}}} \Gamma_{01234} \Gamma_{<\chi>}, \quad (27)$$

where the induced metric component is

$$g_{\chi\chi} = \left( \frac{\tilde{\kappa}}{W} \right)^{2/3} \left( \frac{W}{4} \left( \frac{(\partial_\chi x)^2 + (\partial_\chi y)^2}{y^2} \right) + \frac{\sin^2 \theta}{2} \left( 1 + \frac{(\partial_\chi x)^2}{y^2} \right) \right), \quad (28)$$

and the induced gamma matrix is

$$\Gamma_{<\chi>} = 1_4 \otimes 1_2 \otimes \left( \frac{\tilde{\kappa}}{W} \right)^{1/3} \left[ \frac{\sin \theta}{\sqrt{2}} \left( 1 + \frac{(\partial_\chi x)^2}{y^2} \right) \gamma_9 + \frac{W^{1/2}}{2} \left( \frac{(\partial_\chi x)}{y} \gamma_7 + \frac{(\partial_\chi y)}{y} \gamma_8 \right) \right]. \quad (29)$$

Utilising  $\rho_{01234} = i$ , the requirement for supersymmetry  $\Gamma_{M5} \epsilon = \pm \epsilon$  then reduces to

$$\frac{\tilde{\kappa}^{1/3} W^{-1/3}}{\sqrt{g_{\chi\chi}}} \left[ \frac{\sin \theta}{\sqrt{2}} \left( 1 + \frac{(\partial_\chi x)^2}{y^2} \right) \gamma_9 + \frac{W^{1/2}}{2} \frac{(\partial_\chi x)}{y} \gamma_7 + \frac{W^{1/2}}{2} \frac{(\partial_\chi y)}{y} \gamma_8 \right] \epsilon_0 = \pm \epsilon_0, \quad (30)$$

provided  $\phi_0 = \pi$ . This means that  $\theta = \frac{\pi}{2}$  and  $W = 1$ . In addition, we require the probe to be located at a point on the Riemann surface:

$$\partial_\chi x = \partial_\chi y = 0, \quad (31)$$

so that the terms proportional to  $\gamma_7$  and  $\gamma_8$  disappear. The projection condition  $\gamma_9 \epsilon_0 = \epsilon_0$  also singles out the positive sign above indicating that the probe is an M5-brane as opposed to an anti-M5-brane.

Therefore, the curve  $\mathcal{C}$  is exclusively along the  $\chi$ -direction. As noted in [12], the  $\theta = \frac{\pi}{2}$  condition corresponds to the  $S^2$  shrinking, so the superconformal symmetry  $SU(2) \times U(1)$  symmetry of the background is preserved by this probe. Also, no supersymmetry is broken by this probe.

Now that we have a supersymmetric probe, we may inquire whether it is possible to turn on self-dual  $h$ . As explained earlier, this problem reduces to ensuring

$$a(\Gamma_{012} - \Gamma_{349})\epsilon = 0, \quad (32)$$

where we have defined

$$h = \frac{a}{2} (\mathcal{E}^{012} + \mathcal{E}^{349}). \quad (33)$$

Again using the decomposition (125) and the relationship  $\rho_{01234} = i \Rightarrow \rho_{34} = i\rho_{012}$ , then it is possible to show that this condition is satisfied.



Having verified the kappa-symmetry condition is satisfied, it remains to show that the equations of motion are satisfied. The induced metric may be written

$$ds_{ind}^2 = \frac{\tilde{\kappa}^{2/3}}{2} \left[ \frac{dx_\mu dx^\mu + dr^2}{r^2} + d\chi^2 \right], \quad (34)$$

where we have used Poincaré coordinates.

The RHS of the tensor equation (7) is zero as the background 4-form flux does not pull back to the M5 worldvolume. The tensor equation is then simply

$$G^{mn} \nabla_m H_{npq} = 0. \quad (35)$$

As for scalar equation, the RHS vanishes trivially when  $c \neq 10$ . For the case with  $c = 10$ , the RHS is non-vanishing for general  $\theta$  due to the  $vol_{AdS_5} \wedge d\theta \wedge d\chi$  part of  $H_7$ . However, the coefficient is proportional to  $\cos \theta$ , so it vanishes when it gets pulled back to the worldvolume at  $\theta = \frac{\pi}{2}$ . So, neglecting this case, the scalar equation is simply

$$G^{mn} \nabla_m \mathcal{E}_n^c = 0. \quad (36)$$

This equation is quickly confirmed to be satisfied as it only has one non-trivial component:

$$\nabla_r \mathcal{E}_r^4 = \partial_r \frac{\tilde{\kappa}^{1/3}}{\sqrt{2}r} + \frac{\tilde{\kappa}^{1/3}}{\sqrt{2}r^2} = 0. \quad (37)$$

The ansatz we consider for  $h$  is

$$\begin{aligned} h &= \frac{a}{2} (\mathcal{E}^{012} + \mathcal{E}^{349}) \\ &= \frac{a\tilde{\kappa}}{4\sqrt{2}} \left( \frac{1}{r^3} dt dx_1 dx_2 + \frac{1}{r^2} dx^3 dr d\chi \right), \end{aligned} \quad (38)$$

where  $a$  is a function of  $r$ . Following the treatment in [27],  $H$  may be expressed as

$$H = \frac{a\tilde{\kappa}}{(1+a^2)\sqrt{2}r^3} dt dx_1 dx_2 + \frac{a\tilde{\kappa}}{(1-a^2)\sqrt{2}r^2} dx^3 dr d\chi. \quad (39)$$

As the background 4-form flux doesn't pull back, the Bianchi identity (21) is simply  $dH = 0$ . This means that

$$\frac{a}{(1+a^2)r^3} = \text{constant}. \quad (40)$$

Switching the location of  $dr$  in the flux ansatz above would make this

$$\frac{a}{(1-a^2)r^2} = \text{constant}. \quad (41)$$

Once the Bianchi is satisfied, one may return to the tensor equation. Here  $G^{mn}$  is diagonal and the only term of interest is  $\nabla_r H_{rx_3\chi}$  which is not zero unless  $a$  is a constant. So, we

reach a contradiction and the conclusion is that there is no supersymmetric  $AdS_5$  M5-brane probe with self-dual 3-form  $h$ .

### AdS<sub>4</sub> probes

For  $AdS_4$  probes, a quick look at appendix B reveals that we must mix the spinor  $\psi$  with its conjugate  $\psi^c$ . This is because  $\rho_{0124}\eta_+$  and  $\eta_-$  both have the same eigenvalue under  $\rho^4$ . Thus, we consider

$$\rho_{0124}\psi = c\psi^c, \quad (42)$$

where  $c$  is a constant. The overall effect of this mixing is that the MN Killing spinor gets related to its conjugate through the kappa-symmetry condition.

Adopting the M5 embedding

$$\xi_0 = x_0, \quad \xi_i = x_i (i = 1, 2), \quad \xi_3 = r, \quad \{\xi_4, \xi_5\} \subset M_6, \quad (43)$$

where  $M_6$  denotes the space transverse to  $AdS_5$ , the kappa-symmetry condition may be re-written as

$$c\gamma_7 \frac{\Gamma_{(2)}}{\sqrt{g_2}} (1 + i\sigma_3 \otimes \gamma_{(4)}) \chi_+ e^{-\frac{i}{2}\phi_0\gamma_{10}} e^{i\chi} \epsilon_0 = (1 + i\sigma_3 \otimes \gamma_{(4)}) \chi_+ e^{+\frac{i}{2}\phi_0\gamma_{10}} \epsilon_0, \quad (44)$$

where we have multiplied across by  $e^{i\chi/2}\gamma_7$  and used  $\Gamma_{(2)} \equiv \Gamma_{<\xi_4\xi_5>}$ . One may quickly recognize that a necessary conditions for supersymmetry are

$$[\gamma_7 \Gamma_{(2)}, \sigma_3 \otimes \gamma_{(4)}] = 0, \quad (45)$$

$$\{\gamma_7 \Gamma_{(2)}, \gamma_{10}\} = 0. \quad (46)$$

The latter condition may be ignored if  $\phi_0 = \pi$ . The directions transverse to the  $AdS_5$  space are the product of a two-sphere with a four-dimensional space  $M_6 = S^2 \times M_4$ . Thus, in general,  $\Gamma_{(2)}$  can be a linear combination of two anti-symmetrised gamma matrices, either along  $S^2$ , along  $S^1 \subset S^2$  with a direction in  $M_4$ , or along  $M_4$ . The three possibilities for  $\gamma_7 \Gamma_{(2)}$  are, respectively

$$i\sigma_3 \otimes \gamma_7, \quad \sigma_i \otimes \gamma_7 \gamma_{(4)} \gamma_\mu, \quad 1_2 \otimes \gamma_7 \gamma_{\mu\nu}, \quad (47)$$

where  $i = 1, 2$  and  $\mu, \nu = 7, 8, 9, 10$ . All three choices fail to satisfy (45), so there is no supersymmetric probe with this embedding.

### AdS<sub>3</sub> probes

For  $AdS_3$  probes there is no mixing required between the conjugate MN Killing spinors, so for simplicity, we simply use  $\epsilon = \psi \otimes \xi$  and ignore the conjugate. Referring to (128) and

(129), this allows us to identify  $\eta_+$  as Poincaré and  $\eta_-$  as superconformal Killing spinors, respectively.

After some trial and error, one identifies only one promising candidate embedding

$$\xi_0 = x_0, \quad \xi_1 = x_1, \quad \xi_2 = r, \quad \{\xi_3, \xi_4\} \subset \Sigma_2, \quad \xi_5 = \chi. \quad (48)$$

Using the AdS projection<sup>3</sup>

$$\rho_{014}\psi = -\psi, \quad (49)$$

the kappa-symmetry condition becomes

$$\frac{\Gamma_{<xy\chi>}}{\sqrt{g_3}}\epsilon_0 = \gamma_{789}\epsilon_0 = -i\epsilon_0, \quad (50)$$

where the probe is required to be at  $\theta = \frac{\pi}{2}$  to preserve supersymmetry. The AdS projector  $\rho_{014}\psi = -\psi$  means that  $\rho_{01}\eta_+ = -\eta_+$  and  $\rho_{01}\eta_- = \eta_-$  provided the probe is located at  $x_2 = x_3 = 0$  in  $AdS_5$ . So we can preserve 4 Poincaré and 4 superconformal supersymmetries. As in the case of  $AdS_5$ , the  $SU(2) \times U(1)$  superconformal symmetry is preserved by this probe.

For this supersymmetric probe, the equations of motion can be shown to be satisfied. Here, the induced metric is now

$$ds_{ind}^2 = \frac{\tilde{\kappa}^{2/3}}{2} \left[ \frac{-dx_0^2 + dx_1^2 + dr^2}{r^2} + \frac{1}{2} \frac{(dx^2 + dy^2)}{y^2} + \left( d\chi + \frac{dx}{y} \right)^2 \right]. \quad (51)$$

The scalar equation (6) reduces to two non-trivial components

$$G^{rr}\nabla_r\mathcal{E}_r^4 = G^{yy}\nabla_y\mathcal{E}_y^8 = 0, \quad (52)$$

which may be easily verified to hold. The ansatz for  $h$

$$h = \frac{a\tilde{\kappa}}{4\sqrt{2}} \left( \frac{dx_0 \wedge dx_1 \wedge dr}{r^3} + \frac{dx \wedge dy \wedge d\chi}{2y^2} \right), \quad (53)$$

ensures that both the tensor equation (7) and Bianchi (21) are satisfied when  $a$  is constant.

### AdS<sub>2</sub> probes

As in the case of  $AdS_4$  treated earlier, here we also need to mix MN Killing spinor conjugates. Again, we choose

$$\rho_{04}\psi = c\psi^c, \quad (54)$$

with  $c$  constant. We also adopt the M5 embedding

$$\xi_0 = x_0, \quad \xi_1 = r, \quad \{\xi_2, \dots, \xi_5\} \subset M_6. \quad (55)$$

---

<sup>3</sup>The sign choice here identifies the probe as an M5. An anti-M5 maybe considered by changing sign.

In similar fashion to steps taken before, the kappa-symmetry condition now reads

$$c\gamma_7 \frac{\Gamma_{(4)}}{\sqrt{g_4}} (1 + i\sigma_3 \otimes \gamma_{(4)}) \chi_+ e^{-\frac{i}{2}\phi_0\gamma_{10}} e^{i\chi} \epsilon_0 = (1 + i\sigma_3 \otimes \gamma_{(4)}) \chi_+ e^{+\frac{i}{2}\phi_0\gamma_{10}} \epsilon_0, \quad (56)$$

where now  $\gamma_7 \Gamma_{(4)} \equiv \gamma_7 \Gamma_{<\xi_2 \dots \xi_5>}$  is a linear combination of the building blocks

$$i\sigma_3 \otimes \gamma_7 \gamma_{\mu\nu}, \quad \sigma_i \otimes \gamma_7 \gamma_{(4)} \gamma_{\mu\nu\rho}, \quad 1_2 \otimes \gamma_7 \gamma_{(4)}. \quad (57)$$

These correspond to the probe wrapping  $S^2$ , wrapping  $S^1 \subset S^2$ , and the probe not wrapping  $S^2$ , respectively. Before we can even consider talking about projection conditions, a necessary condition for supersymmetry is that  $\gamma_7 \Gamma_{(4)}$  commutes with  $\sigma_3 \otimes \gamma_{(4)}$ . In much the same way as for  $AdS_2$ , this condition is not satisfied, thus ruling out the possibility of a simple supersymmetric  $AdS_2$ .

### 3.2 Other supersymmetric probes

In this section, we repeat the steps of the last section for probes at a fixed value of  $r$ . We catalogue the possibilities below. In general, one may consider change the embedding along a spatial direction of  $AdS_5$  into the radial direction  $r$ , getting new M5-brane configurations. We anticipate that those non-supersymmetric configurations are also solutions to the equations of motion. For  $M_4$ , these branes should correspond to domain walls on the field theory side. It would be interesting to eventually study how the gauge theory changes when one crosses the domain wall.

#### $M_4$ probes

Here again we have no mixing between the MN Killing spinor and its conjugate, so we opt to just work with  $\epsilon = \psi \otimes \xi$ . Introducing the projector

$$\rho_{0123}\psi = \pm i\psi, \quad (58)$$

we adopt the following embedding for the M5-brane

$$\xi_0 = x_0, \quad \xi_i = x_i \ (i = 1, 2, 3), \quad \{\xi_4, \xi_5\} \subset M_6. \quad (59)$$

After some preliminary trial and error using the background projectors (3), one finds that there are two promising candidates for embeddings:

$$\{\xi_4, \xi_5\} \subset \Sigma_2 \text{ and } \{\xi_4, \xi_5\} \subset S^2. \quad (60)$$

For the first embedding, we take  $\chi$  to be a function of  $x, y$  i.e.  $\chi \equiv \chi(x, y)$ . We also neglect  $\theta$  as there is no  $\gamma_{10}$  projector acting on the MN Killing spinors. Making use of

$\rho_{0123}\psi = i\psi$ , the kappa symmetry condition becomes

$$\frac{i\Gamma_{<xy>}}{\sqrt{g_2}}\epsilon_0 = \epsilon_0, \quad (61)$$

where  $g_2$  denotes the induced metric. The induced gamma matrices are

$$\begin{aligned} \Gamma_{<x>} &= \tilde{\kappa}^{1/3} \left[ \frac{W^{1/6}}{2y} \gamma_7 + \frac{\sin \theta W^{-1/3}}{\sqrt{2}y} \gamma_9 + \partial_x \chi \frac{\sin \theta W^{-1/3}}{\sqrt{2}} \gamma_9 \right], \\ \Gamma_{<y>} &= \tilde{\kappa}^{1/3} \left[ \frac{W^{1/6}}{2y} \gamma_8 + \partial_x \chi \frac{\sin \theta W^{-1/3}}{\sqrt{2}} \gamma_9 \right]. \end{aligned} \quad (62)$$

Setting  $\theta = 0$ , we find that this configuration is supersymmetric with 4 Poincaré supersymmetries preserved. One may also switch on  $h$ -field of the form

$$h = \frac{a}{2}(\mathcal{E}^{012} + \mathcal{E}^{378}). \quad (63)$$

In the second case, the kappa symmetry condition becomes

$$\sigma_3 \chi_+ = \pm \chi_+. \quad (64)$$

On top of the projector (58), this breaks supersymmetry further. So we are left with 2 Poincaré supersymmetries. There is no condition on  $\theta$  and provided we avoid  $\theta = \frac{\pi}{2}$  where the  $S^2$  shrinks. It is easy to show that the above projector also supports a self-dual  $h$ -field like

$$h = \frac{a}{2}(\mathcal{E}^{012} + \mathcal{E}^{378}). \quad (65)$$

The equations of motion give more constraints. One of the scalar equations require  $\theta$  should be zero, for both vanishing  $h$  and the non-zero  $h$ -field given above.

### **M<sub>3</sub> probes**

We find there is no simple supersymmetric probe obtained by mixing  $\psi$  with its conjugate  $\psi^c$ . The difficulties presented in treating the relevant factor  $e^{i\chi}$  will also be found in an  $M_1$  probe, which we choose not to pursue.

### **M<sub>2</sub> probes**

As in the case of  $M_4$ , we confine our attention to  $\epsilon = \psi \otimes \xi$ . Introducing the projection condition

$$\rho_{01}\psi = \pm \psi, \quad (66)$$

we consider an embedding of the form

$$\xi_0 = x_0, \quad \xi_1 = x_1 \quad \{\xi_2, \xi_3\} \subset S^2, \quad \{\xi_4, \xi_5\} \subset \Sigma_2. \quad (67)$$

This embedding preserves supersymmetry provided we allow for the additional projector

$$\sigma_3 \chi_+ = \pm \chi_+. \quad (68)$$

As  $i\gamma_{78}$  commutes with  $\gamma_{10}$ , supersymmetry does not pick out a specific angle for  $\theta$  provided we avoid the  $S^2$  shrinking. In total 2 Poincaré supersymmetries are preserved. This ansatz satisfies the M5 equations of motion when  $\theta = 0$ . We can get non-BPS brane with an  $AdS_2$  factor from the brane with an  $\mathcal{M}_2$  factor. This non-BPS brane should be dual to some one-dimensional object in the field theory side.

## 4 More non-supersymmetric probes

Inspired by some M5-branes in  $AdS_7 \times S^4$  [26, 27], in this section we consider fibrations of  $AdS_5$  that may give rise to loop operators ( $AdS_2 \times S^1 \subset AdS_5$ ,  $AdS_2 \times S^2 \subset AdS_5$ ) and surface operators ( $AdS_3 \times S^1 \subset AdS_5$ ). We begin by analysing the equations of motion for the  $AdS_3$  M5-brane probes which turn out to be instructive in making comments about the  $AdS_2$  case.

We begin with an M5 whose worldvolume is  $AdS_3 \times S^1 \times \Sigma_2$  with  $AdS_3 \times S^1$  in  $AdS_5$ . We write  $ds_{AdS_5}^2$  as:

$$ds_{AdS_5}^2 = \cosh^2 \rho (-\cosh^2 \zeta d\tau^2 + d\zeta^2 + \sinh^2 \zeta d\varphi^2) + d\rho^2 + \sinh^2 \rho d\alpha^2. \quad (69)$$

The vielbein of the background geometry is:

$$E^0 = \frac{\tilde{\kappa}^{1/3} W^{1/6}}{\sqrt{2}} \cosh \rho \cosh \zeta d\tau, \quad E^1 = \frac{\tilde{\kappa}^{1/3} W^{1/6}}{\sqrt{2}} \cosh \rho d\zeta, \quad (70)$$

$$E^2 = \frac{\tilde{\kappa}^{1/3} W^{1/6}}{\sqrt{2}} \cosh \rho \sinh \zeta d\varphi, \quad E^3 = \frac{\tilde{\kappa}^{1/3} W^{1/6}}{\sqrt{2}} d\rho, \quad (71)$$

$$E^4 = \frac{\tilde{\kappa}^{1/3} W^{1/6}}{\sqrt{2}} \sinh \rho d\alpha, \quad E^5 = \frac{\tilde{\kappa}^{1/3} W^{-1/3} \cos \theta}{2} d\phi_1, \quad (72)$$

$$E^6 = \frac{\tilde{\kappa}^{1/3} W^{-1/3} \cos \theta \sin \phi_1}{2} d\phi_2, \quad E^7 = \frac{\tilde{\kappa}^{1/3} W^{1/6}}{2} \frac{dx}{y}, \quad (73)$$

$$E^8 = \frac{\tilde{\kappa}^{1/3} W^{1/6}}{2} \frac{dy}{y}, \quad E^9 = \frac{\tilde{\kappa}^{1/3} W^{-1/3} \sin \theta}{\sqrt{2}} (d\chi + \frac{dx}{y}), \quad (74)$$

$$E^{10} = \frac{\tilde{\kappa}^{1/3} W^{1/6}}{2} d\theta. \quad (75)$$

The ansatz of the M5-brane is:

$$\xi_0 = \tau, \xi_1 = \zeta, \xi_2 = \varphi, \quad (76)$$

$$\xi_3 = \alpha, \xi_4 = x, \xi_5 = y, \quad (77)$$

$$\rho = \rho_0, \theta = \theta_0. \quad (78)$$

The induced metric is:

$$\begin{aligned} d\tilde{s}^2 &= \tilde{\kappa}^{2/3} \frac{1}{2} W_0^{1/3} (-\cosh^2 \rho_0 \cosh^2 \zeta d\tau^2 + \cosh^2 \rho_0 d\zeta^2 + \cosh^2 \rho_0 \sinh^2 \zeta d\varphi^2) \\ &+ \tilde{\kappa}^{2/3} \left( \frac{1}{2} W_0^{1/3} \sinh^2 \rho_0 d\alpha^2 + \frac{W_0^{1/3} (dx^2 + dy^2)}{4y^2} + \frac{1}{2W_0^{2/3}} \sin^2 \theta_0 \frac{dx^2}{y^2} \right), \end{aligned} \quad (79)$$

where

$$W_0 = 1 + \cos^2 \theta_0. \quad (80)$$

The ansatz for  $h$  is:

$$h = \frac{a}{2} \kappa \left( \frac{W_0^{1/2}}{2\sqrt{2}} \cosh^3 \rho_0 \sinh \zeta \cosh \zeta d\tau d\zeta d\varphi + \frac{W_0^{1/2}}{4\sqrt{2}} \frac{\sinh \rho_0}{y^2} d\alpha dx dy \right). \quad (81)$$

From this we can get:

$$k_m^n = \begin{pmatrix} -\frac{a^2}{2} I_3 & 0 \\ 0 & \frac{a^2}{2} I_3 \end{pmatrix}, \quad (82)$$

$$Tr k^2 = \frac{3}{2} a^4, Q = 1 - a^4, \quad (83)$$

$$H = 2a\kappa^{-1} \left( \frac{W_0^{1/2}}{2\sqrt{2}(1+a^2)} \cosh^3 \rho_0 \sinh \zeta \cosh \zeta d\tau \omega d\zeta \omega d\varphi \right. \quad (84)$$

$$\left. + \frac{W_0^{1/2}}{4\sqrt{2}(1-a^2)} \frac{\sinh \rho_0}{y^2} d\alpha dx dy \right) \quad (85)$$

From  $\underline{H}_4 = 0$ , we get that  $dH = \underline{H}_4$  is satisfied if  $a$  is constant.

The results for  $G^{mn}$  is:

$$G^{\tau\tau} = (1+a^2)^2 g^{\tau\tau}, G^{\zeta\zeta} = (1+a^2)^2 g^{\zeta\zeta}, G^{\varphi\varphi} = (1+a^2)^2 g^{\varphi\varphi} \quad (86)$$

$$G^{\alpha\alpha} = (1-a^2)^2 g^{\alpha\alpha}, G^{xx} = (1-a^2)^2 g^{xx}, G^{yy} = (1-a^2)^2 g^{yy}. \quad (87)$$

The tensor equations now become:

$$G^{mn} \nabla_m H_{npq} = 0. \quad (88)$$

The above ansatz satisfy these equations when  $\theta_0 = 0$ .

The scalar equations become:

$$G^{mn} \nabla_m \mathcal{E}_n^c = 0, \quad (89)$$

for  $c \neq 3$ . When  $\theta_0 = 0$ , The equations for  $c \neq 3$  are satisfied automatically. For the equation with  $c = 3$ , the RHS is non zero due to  $H_7$ . Among three terms in  $H_4$ , only the following contributes:

$$-\frac{2\sqrt{2}(3 + \cos^2 \theta)}{\tilde{\kappa}^{1/3}W^{7/6}}e^{\frac{789(10)}{0123456}}, \quad (90)$$

whose Hodge dual is:

$$-\frac{2\sqrt{2}(3 + \cos^2 \theta)}{\tilde{\kappa}^{1/3}W^{7/6}}e^{\frac{0123456}{789(10)}}, \quad (91)$$

The scalar equation with  $c = 3$  is:

$$\frac{\sqrt{2}}{\tilde{\kappa}^{1/3}W_0^{1/6}}(3(1 + a^2)^2 \frac{\sinh \rho}{\cosh \rho} + (1 - a^2)^2 \frac{\cosh \rho}{\sinh \rho}) = \frac{2\sqrt{2}(3 + \cos^2 \theta_0)(1 - a^4)}{\tilde{\kappa}^{1/3}W_0^{7/6}} \quad (92)$$

By using  $\theta_0 = 0$ , we get

$$3(1 + a^2)^2 \frac{\sinh \rho}{\cosh \rho} + (1 - a^2)^2 \frac{\cosh \rho}{\sinh \rho} = 4(1 - a^4). \quad (93)$$

By the mean value inequality, the absolute value of the LHS is not less than

$$2\sqrt{3}|1 - a^4|, \quad (94)$$

so this equation has real root when  $|a| \leq 1$ .

If we consider M5 with  $AdS_2 \times S^2 \times \Sigma_2$  with  $AdS_2 \times S^2$  inside  $AdS_5$ , it seems that the only change is the scalar equation with  $c = 3$ , we get

$$2(1 + a^2)^2 \frac{\sinh \rho}{\cosh \rho} + 2(1 - a^2)^2 \frac{\cosh \rho}{\sinh \rho} = 4(1 - a^4). \quad (95)$$

We have similar results as above. We have also verified that there are no M5-brane solutions with factor  $AdS_2 \times S^1 \subset AdS_5$ .

## 4.1 Supersymmetry

In this subsection we identify the conditions for supersymmetry to be preserved by the above probes. We begin with  $AdS_3$  by solving the Killing spinor equation for an  $AdS_3 \times S^1$  fibration of  $AdS_5$ . The result of the analysis stipulates that the probe has to be located at  $\rho = \infty$  for supersymmetry to be preserved. Therefore, we validate our claim that the above probe located at finite  $\rho$  is non-supersymmetric. We also sketch the calculation for  $AdS_2 \times S^2$ .

In each case we employ the same fibration with vielbein

$$E^a = \cosh \rho \bar{E}_{AdS_n}^a, \quad e^n = d\rho, \quad e^\alpha = \sinh \rho \bar{E}_{S^{4-n}}^\alpha, \quad (96)$$



where  $a = 0, \dots, n-1$  and  $\alpha = n+1, \dots, 4$ .

For  $AdS_3 \times S^1$ , we introduce the following decomposition for the  $AdS_5$  gamma matrices

$$\rho_a = \tau_a \otimes \sigma_3, \quad \rho_3 = 1 \otimes \sigma_1, \quad \rho_4 = 1 \otimes \sigma_2, \quad (97)$$

where

$$\{\tau_a, \tau_b\} = 2\eta^{ab}, \quad (98)$$

and  $\sigma_i$  denote the Pauli matrices. By further writing the  $AdS_5$  spinor,  $\psi$  as the product  $\psi \equiv \chi \otimes \xi$ , the Killing spinor equation on  $AdS_5$  become

$$\frac{1}{\cosh \rho} \xi + \frac{\sinh \rho}{\cosh \rho} i \sigma_2 \xi = \sigma_3 \xi, \quad (99)$$

$$\partial_\rho \xi = \frac{\sigma_1}{2} \xi, \quad (100)$$

$$\partial_\alpha \xi + \frac{i \sigma_3}{2} \cosh \rho \xi = \frac{\sigma_2}{2} \sinh \rho \xi. \quad (101)$$

Here, we have used the Killing spinor equation on  $AdS_3$ :  $\nabla_a \chi = \frac{\tau_a}{2} \chi$ . Writing the two-dimensional complex spinor as

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}, \quad (102)$$

(99) tells us that the two components are not independent:

$$\sinh \left( \frac{\rho}{2} \right) \xi_1 = \cosh \left( \frac{\rho}{2} \right) \xi_2. \quad (103)$$

Then solving (100) and (101), we find that the solution is of the final form for the  $AdS_5$  Killing spinor is

$$\psi \equiv \chi \otimes e^{-i\alpha/2} \begin{pmatrix} \cosh \left( \frac{\rho}{2} \right) \\ \sinh \left( \frac{\rho}{2} \right) \end{pmatrix}, \quad (104)$$

where  $\alpha$  denotes one of the angles of the  $S^2$ .

We can now determine the supersymmetry condition on a flat probe in the  $AdS_3 \times S^1$  directions. The projection condition  $\rho^{0124} \psi = i\psi$  implies

$$\sinh \rho = \cosh \rho. \quad (105)$$

So, supersymmetry can be preserved at  $\rho = \infty$ .

For  $AdS_2 \times S^2$  the calculation runs as follows. We introduce the gamma matrix decomposition

$$\rho_a = \tau_a \otimes 1, \quad \rho_2 = \tau_3 \otimes \sigma_3, \quad \rho_\alpha = \tau_3 \otimes \sigma_\alpha, \quad (106)$$

and the following decomposition for the spinor

$$\psi = f^{AB} \chi_A \otimes \xi_B, \quad (107)$$

where the  $AdS_2$  spinor  $\chi_A$  and the  $S^2$  spinor  $\xi_B$  satisfy the Killing spinor equations

$$\begin{aligned}\nabla_a \chi_{\pm} &= \pm \frac{1}{2} \tau_3 \tau_a \chi_{\pm}, \\ \nabla_a \xi_{\pm} &= \pm \frac{1}{2} \sigma_a \xi_{\pm}.\end{aligned}\tag{108}$$

The components of the respective spinors are also related via

$$\tau_3 \chi_+ = \chi_-, \quad \sigma_3 \xi_+ = \xi_-.\tag{109}$$

With this set-up, placing  $\psi$  directly into the  $AdS_5$  Killing spinor equation, one arrives at the following form of the Killing spinor

$$\psi = \cosh(\frac{\rho}{2}) \chi_+ \otimes \xi_+ + \sinh(\frac{\rho}{2}) \chi_- \otimes \xi_-,\tag{110}$$

with  $f^{+-} = f^{-+} = 0$ . One can then readily verify that the probe projector  $\rho_{0134} \psi = i\psi$  can only be solved when  $\rho = \infty$ .

## 5 Conclusion

In this paper, motivated by the recent advances in our understanding of  $\mathcal{N} = 2$  SCFTs, we have attempted to identify simple probe M5-branes in the MN background preserving supersymmetry. In addition to the known  $AdS_5 \times S^1$  probe, our analysis identified an  $AdS_3 \times \Sigma_2 \times S^1$  counterpart embedding that breaks supersymmetry further. As the M5 also wraps the Riemann surface in this case, its interpretation as a two-dimensional object in the dual field theory is still unclear. In addition, one unusual aspect of our study is the realization that the  $AdS_5 \times S^1$  M5-brane probe does not support a self-dual  $h$ -field. It is ruled out by the equations of motion.

We have also identified other BPS and non-BPS probes that should correspond to some non-local objects in the dual theory. We hope to study these objects more in future work and provide some better illumination of their properties. As a future direction, one can immediately imagine generalizing the probes we have identified in MN to the more general class of LLM geometries. Another open avenue concerns backreacting the probe branes in the literature along the lines of work pioneered by Lunin for  $AdS_4 \times S^7$  and  $AdS_7 \times S^4$ .

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## A Maldacena-Núñez background

The ansatz for the d=7 metric appearing in [14] is

$$ds_7^2 = e^{2f(r)}(-dt^2 + dx_i^2 + dr^2) + \frac{e^{2g(r)}}{y^2}(dx^2 + dy^2). \quad (111)$$

From the supersymmetry variations in [14], the general solution where  $x, y$  define a Hyperbolic space may be written

$$\begin{aligned} e^{5\lambda} &= \frac{e^{2\rho} + \frac{1}{2} + C_1 e^{-2\rho}}{e^{2\rho} + \frac{1}{4}}, \\ e^{2g} &= e^\lambda(e^{2\rho} + \frac{1}{4}), \\ e^{2f} &= C_2 e^{2\rho} e^\lambda, \\ e^{2f} \left(\frac{dr}{d\rho}\right)^2 &= e^{-4\lambda}. \end{aligned} \quad (112)$$

Here  $\rho \rightarrow \infty$  corresponds to the boundary of  $AdS_7$ ,  $C_2$  is a trivial integration constant that may be absorbed by a volume rescaling, and when  $C_1 = 0$ , the solution interpolates between  $AdS_7$  and  $AdS_5 \times \Sigma$ . The solution in the IR has the fixed point values

$$e^{5\lambda} = 2, \quad e^{2g-\lambda} = \frac{1}{4}, \quad e^{f+2\lambda} = \frac{1}{r}, \quad (113)$$

making the d=7 metric

$$ds_7^2 = e^\lambda \left[ \frac{1}{2} ds_{AdS_5}^2 + \frac{1}{4} \frac{(dx^2 + dy^2)}{y^2} \right]. \quad (114)$$

Uplifting this solution to d=11 makes use of section 4 from [36], and in particular, the following formula

$$\begin{aligned} ds_{11}^2 &= \tilde{\Delta}^{1/3} ds_7^2 + g^{-2} \tilde{\Delta}^{-2/3} \left( X_0^{-1} d\mu_0^2 + \sum_{i=1}^2 X_i^{-1} (d\mu_i^2 + \mu_i^2 (d\phi_i + gA^i)^2) \right), \\ *_{11} H_4 &= 2g \sum_{\alpha=0}^2 \left( X_\alpha^2 \mu_\alpha^2 - \tilde{\Delta} X_\alpha \right) vol_7 + g \tilde{\Delta} X_0 vol_7 + \frac{1}{2g} \sum_{\alpha=0}^2 X_\alpha^{-1} *_7 dX_\alpha \wedge d(\mu_\alpha^2) \\ &+ \frac{1}{2g^2} \sum_{i=1}^2 X_i^{-2} d(\mu_i^2) \wedge (d\phi_i + gA^i) \wedge *_7 F^i, \end{aligned} \quad (115)$$

where  $*_7$  and  $vol_7$  are the Hodge dual and the volume form with respect to the metric  $ds_7^2$  and  $*_{11}$  is the Hodge dual of the uplifted metric. In addition, we have the following relationships:

$$X_0 \equiv (X_1 X_2)^{-2}, \quad \tilde{\Delta} = \sum_{\alpha=0}^2 X_\alpha \mu_\alpha^2, \quad \sum_{\alpha=0}^2 \mu_\alpha^2 = 1. \quad (116)$$

Making contact between the two actions, i.e. (89) of [14] and (4.6) of [36], means adopting the following identifications

$$\begin{aligned} g &= 2, \quad X_0 = X_2 = e^{2\lambda}, \quad X_1 = e^{-3\lambda}, \quad 2A_{MN}^i = A^i \\ \mu_0 &= \cos \theta \cos \psi, \quad \mu_1 = \sin \theta, \quad \mu_2 = \cos \theta \sin \psi. \end{aligned} \quad (117)$$

With these identifications

$$\tilde{\Delta} = e^{-3\lambda}(1 + \cos^2 \theta) = e^{-3\lambda}W. \quad (118)$$

We also obtain the metric

$$\begin{aligned} ds_{11}^2 &= \frac{1}{2}W^{1/3}ds_{AdS_5}^2 + \frac{W^{-2/3}}{4} \left[ W \frac{(dx^2 + dy^2)}{y^2} \right. \\ &\quad \left. + Wd\theta^2 + \cos^2 \theta (d\psi^2 + \sin^2 \psi d\phi_2^2) + 2 \sin^2 \theta \left( d\phi_1 + \frac{dx}{y} \right)^2 \right], \end{aligned} \quad (119)$$

where we have used  $A_{MN}^1 = y/4dx$ . One may also determine the fluxes from the formula above. The term

$$2g \sum_{\alpha=0}^2 \left( X_\alpha^2 \mu_\alpha^2 - \tilde{\Delta} X_\alpha \right) vol_7 + g \tilde{\Delta} X_0 vol_7 \quad (120)$$

gives the following contribution to  $H_4$ :

$$-\frac{1}{4W^2} [3 + \cos^2 \theta] \sin \theta \cos^2 \theta d\theta \left( d\phi_1 + \frac{dx}{y} \right) \sin \psi d\psi d\phi_2. \quad (121)$$

The next term is zero and the last term

$$\frac{1}{2g^2} \sum_{i=1}^2 X_i^{-2} d(\mu_i^2) \wedge (d\phi_i + gA^i) \wedge *_7 F^i, \quad (122)$$

becomes

$$\frac{1}{4} \frac{\cos^3 \theta}{W} \frac{dxdy}{y^2} \sin \psi d\psi d\phi_2. \quad (123)$$

Up to relabeling of coordinates, this is the form of the solution appearing in the introduction.

## B MN Killing spinors

Parallel to the treatment in [13], we introduce a decomposition for the D=11 gamma matrices satisfying

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}, \quad M, N = 0, \dots, 10. \quad (124)$$

These may be re-expressed in terms of lower-dimensional gamma matrices as

$$\begin{aligned} \Gamma^a &= \rho^a \otimes \sigma_3 \otimes \gamma_{(4)}, \\ \Gamma^i &= 1_4 \otimes \sigma_i \otimes \gamma_{(4)}, \\ \Gamma^\mu &= 1_4 \otimes 1_2 \otimes \gamma_\mu, \end{aligned} \quad (125)$$

where  $a = 0, \dots, 4$  denote  $AdS_5$  directions,  $i = 1, 2$  denote directions along the  $S^2$  and  $\mu = 7, \dots, 10$  label gamma matrices along the remaining  $(x, y, \chi, \theta)$  directions, respectively.  $\gamma_{(4)}$  is simply the product  $\gamma_{78910}$  and we adopt the sign choice  $\Gamma_{012345678910} = -1$ . Consequently, this implies that  $\rho_{01234} = i$ .

Throughout this paper, we will make use of the explicit construction of Killing spinors on AdS spacetimes appearing in [37]. Writing the  $AdS_5$  metric as

$$ds_{AdS_5}^2 = \frac{1}{r^2} (dx_\mu dx^\mu + dr^2), \quad (126)$$

the solutions to the Killing spinor equation

$$D_a \psi = \frac{1}{2} \rho_a \psi, \quad (127)$$

may be expressed as

$$\psi_+ = r^{-1/2} \eta_+, \quad (128)$$

$$\psi_- = r^{1/2} \eta_- + r^{-1/2} x^\alpha \rho_\alpha \eta_-, \quad (129)$$

where  $a = 0, \dots, 4$  labels the  $AdS_5$  coordinates including  $r$ , and  $\alpha = 0, \dots, 3$  omits  $r$ . The constant spinors  $\eta_+, \eta_-$  correspond to Poincaré and superconformal Killing spinors and are subject to the additional projection condition [37]

$$\rho_r \eta_\pm = \pm \eta_\pm. \quad (130)$$

Note that replacing  $\psi$  with its conjugate  $\psi^c$  results in a sign change in the Killing spinor equation (127). This knock-on effect of this change is that  $\eta_+$  and  $\eta_-$  get interchanged in the solution.

From here on, we write down a general expression for a Killing spinor preserved by the MN background as

$$\epsilon = \psi \otimes \xi + \psi^c \otimes \xi^c, \quad (131)$$

where we just focus on the Poincaré Killing spinors

$$\psi = r^{-1/2}\eta_+, \quad \psi^c = r^{-1/2}\eta_-. \quad (132)$$

The rest of this section concerns the identification of  $\xi$  and  $\xi^c$  from the Killing spinor equation for the MN background.

We begin by examining the eleven-dimensional Killing spinor equation

$$\nabla_m \eta + \frac{1}{288} [\Gamma_m^{npqr} - 8\delta_m^n \Gamma^{pqr}] H_{4npqr} \eta = 0. \quad (133)$$

As in LLM [13], the analytically continued solution may be reduced on  $AdS_5$ , then  $S^2$ , before the differential constraints on the remaining four-dimensional space may be extracted. Within the framework of LLM we can incorporate the two sign choices in the  $AdS_5$  Killing spinor equation as:

$$D_a \Psi = (ib) \frac{i}{2} \rho_a \Psi, \quad (134)$$

where  $b = -1$  for  $\Psi = \psi$  and  $b = 1$  for  $\Psi = \psi^c$ .

Introducing the gamma matrix decomposition introduced earlier (125) and following the steps as outlined in appendix F of LLM, one arrives at the following equations

$$\left[ \gamma^\mu \partial_\mu \lambda + \frac{a}{12} e^{-3\lambda-2A} \gamma_{(4)} \gamma^{\mu\nu} F_{\mu\nu}^{(2)} + iabm \right] \epsilon = 0, \quad (135)$$

$$\left[ ie^{-A} \gamma_{(4)} + \gamma^\mu \partial_\mu A - \frac{a}{4} e^{-3\lambda-2A} \gamma_{(4)} \gamma^{\mu\nu} F_{\mu\nu}^{(2)} - iabm \right] \epsilon = 0, \quad (136)$$

$$\left[ \nabla_\mu - i \frac{abm}{2} \gamma_\mu - \frac{a}{4} e^{-3\lambda-2A} F_{\mu\nu}^{(2)} \gamma^\nu \gamma_{(4)} \right] \epsilon = 0, \quad (137)$$

with  $a = \pm 1$  and  $F^{(2)}$  defined in terms of the four-form flux by

$$H_4 = F^{(2)} \wedge d^2 \Omega. \quad (138)$$

As in LLM combining (135) and (136) to remove the  $F^{(2)}$  terms, we find the condition

$$\partial_\mu (A + 3\lambda) \gamma^\mu \epsilon + ie^{-A} \gamma_{(4)} \epsilon + 2iabm \epsilon = 0. \quad (139)$$

Taking  $m = \frac{1}{2}$ , the solution to this equation is

$$\epsilon = e^{-iab\gamma_{10}\phi_0/2} \epsilon_0, \quad (140)$$

where

$$\sin \phi_0 = \frac{\sqrt{2} \cos \theta}{\sqrt{W}}, \quad \cos \phi_0 = -\frac{\sin \theta}{\sqrt{W}}, \quad (141)$$

and  $\epsilon_0$  satisfies the projection condition

$$(i\gamma_{10}\gamma_{(4)} + 1)\epsilon_0 = 0. \quad (142)$$

Returning then to (135), we may determine the second projection condition from the requirement that it is satisfied by the MN solution. This amounts to the following being satisfied

$$\left[ -\frac{\sin \theta \cos \theta}{3\sqrt{2}W} \gamma_{10} - \frac{a \cos \theta}{3\sqrt{2}W} \gamma_{910} - \frac{a(3 + \cos^2 \theta)}{6W} \gamma_{78} + iabm \right] \epsilon = 0. \quad (143)$$

Using (140),  $m = \frac{1}{2}$ ,  $\gamma_9 \epsilon_0 = \alpha \epsilon_0$ <sup>4</sup> where  $\alpha = \pm 1$ , we may expand this expression to get two parts, one proportional to the identity and the other proportional to  $\gamma_{10}$ :

$$\begin{aligned} \gamma_{10} \left[ -\frac{sc}{3\sqrt{2}W} \left(1 - \frac{s}{\sqrt{W}}\right) + \frac{c}{\sqrt{2}W} + \alpha a \frac{c}{3\sqrt{2}W} \left(1 - \frac{s}{\sqrt{W}}\right) + \alpha b \frac{(3 + c^2)c}{3\sqrt{2}W\sqrt{W}} \right] \epsilon_0 = 0 \\ 1_4 \left[ ab \frac{sc^2}{3W\sqrt{W}} + \frac{1}{2} ab \left(1 - \frac{s}{\sqrt{W}}\right) + \alpha b \frac{c^2}{3W} + \alpha a \frac{(3 + c^2)}{6W} \left(1 - \frac{s}{\sqrt{W}}\right) \right] \epsilon_0 = 0. \end{aligned} \quad (144)$$

Here we have employed the shorthand  $c \equiv \cos \theta$ ,  $s \equiv \sin \theta$  to compress these expressions. These are satisfied provided

$$a = b = -\alpha, \quad \text{with } a^2 = 1. \quad (145)$$

The final part of the Killing spinor may be determined by solving (137) directly to determine the functional dependence of  $\epsilon_0$ . From the projectors  $i\gamma_{78}\epsilon_0 = \alpha\epsilon_0$  and  $\gamma_9\epsilon_0 = \alpha\epsilon_0$ , we can track the dependence on the sign  $\alpha$ . After examining (137) one determines that

$$\partial_\theta \epsilon_0 = \partial_x \epsilon_0 = \partial_y \epsilon_0 = 0. \quad (146)$$

The dependence on  $\chi$  may easily be determined from the Killing spinor equation in the  $x$  or  $\chi$  directions. For ease of illustration we focus on the  $x$  direction. After a small calculation this becomes

$$\begin{aligned} \left[ \frac{y}{\sqrt{2}} \left( \partial_x - \frac{\partial_\chi}{y} \right) + \frac{1}{2} \left( -\frac{1}{\sqrt{2}} \gamma_{78} - \frac{\sin \theta}{2\sqrt{W}} \gamma_{98} \right) \right. \\ \left. - \frac{iab}{4} \gamma_7 + \frac{a\sqrt{2} \cos \theta}{4\sqrt{W}} \gamma_{789} \right] e^{-\frac{i}{2} ab \gamma_{10} \phi_0} \epsilon_0(\chi) = 0. \end{aligned} \quad (147)$$

In this expression, after using the projectors, there are terms proportional to  $1_4$ ,  $\gamma_8$  and  $\gamma_{810}$ . The latter two cancel independently of their own accord, but the term proportional to the identity becomes

$$\left[ \partial_\chi + \frac{1}{2} \gamma_{78} \right] \epsilon_0(\chi) = 0. \quad (148)$$

Using  $i\gamma_{78}\epsilon_0 = \alpha\epsilon_0$ ,  $\epsilon_0(\chi)$  may be written simply as

$$\epsilon_0(\chi) = e^{\frac{i}{2} \alpha \chi} \tilde{\epsilon}_0, \quad (149)$$

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<sup>4</sup>Implying  $i\gamma_{78}\epsilon_0 = \alpha\epsilon_0$  from (142).

where  $\tilde{\epsilon}_0$  is a constant spinor. We can therefore confirm the form of the Killing spinors for MN appearing in the text (3). The extra  $\gamma_7$  has been introduced to ensure that the projection conditions are correct.

## C Some connections

In this appendix, we list the connections appear in section 4. The nontrivial independent components of the Levi-Civita connection for the induced metric eq. (79) are:

$$\Gamma_{\tau\tau}^{\zeta} = -\Gamma_{\varphi\varphi}^{\zeta} = \cosh \zeta \sinh \zeta, \quad (150)$$

$$\Gamma_{\tau\zeta}^{\tau} = \frac{\sinh \zeta}{\cosh \zeta}, \quad (151)$$

$$\Gamma_{\varphi\zeta}^{\varphi} = \frac{\cosh \zeta}{\sinh \zeta}, \quad (152)$$

$$\Gamma_{yx}^x = \Gamma_{yy}^y = -\frac{1}{y}, \quad (153)$$

$$\Gamma_{xx}^y = \frac{1}{y} \left( 1 + \frac{2 \sin^2 \theta_0}{W_0} \right). \quad (154)$$

Some of the nonzero independent components of the spin connection with respect to the vielbeins in that section are:

$$\omega_{\underline{00}}^1 = \frac{\sqrt{2} \sinh \zeta}{\tilde{\kappa}^{1/3} W^{1/6} \cosh \rho \cosh \zeta}, \quad (155)$$

$$\omega_{\underline{22}}^1 = -\frac{\sqrt{2} \cosh \zeta}{\tilde{\kappa}^{1/3} W^{1/6} \cosh \rho \sinh \zeta}, \quad (156)$$

$$\omega_{\underline{00}}^3 = -\omega_{\underline{11}}^3 = -\omega_{\underline{22}}^3 = \frac{\sqrt{2} \sinh \rho}{\tilde{\kappa}^{1/3} W^{1/6} \cosh \rho}, \quad (157)$$

$$\omega_{\underline{44}}^3 = -\frac{\sqrt{2} \cosh \rho}{\tilde{\kappa}^{1/3} W^{1/6} \sinh \rho}, \quad (158)$$

$$\omega_{\underline{77}}^8 = \frac{2}{\tilde{\kappa}^{1/3} W^{1/6}}, \quad (159)$$

$$\omega_{\underline{00}}^{10} = -\omega_{\underline{ii}}^{10} = -\frac{2 \sin \theta \cos \theta}{3 \tilde{\kappa}^{1/3} W^{1/6} (1 + \cos^2 \theta)}, i = 1, \dots, 4, 7, 8, \quad (160)$$

$$\omega_{\underline{55}}^{10} = \omega_{\underline{66}}^{10} = \frac{2}{\tilde{\kappa}^{1/3} W^{1/6}} \left( \frac{\sin \theta}{\cos \theta} - \frac{2 \sin \theta \cos \theta}{3(1 + \cos^2 \theta)} \right), \quad (161)$$

$$\omega_{\underline{99}}^{10} = \frac{2}{\tilde{\kappa}^{1/3} W^{1/6}} \left( -\frac{\cos \theta}{\sin \theta} - \frac{2 \sin \theta \cos \theta}{3(1 + \cos^2 \theta)} \right). \quad (162)$$

The remaining non-vanishing components are:

$$\omega_{\underline{66}}^5, \omega_{\underline{98}}^7, \omega_{\underline{89}}^7, \omega_{\underline{79}}^8, \quad (163)$$

but they are not needed in the computations in this paper.



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